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INTEGRALS IN THE TOPLOSKY RIB FORMULATION

G. A. Lengua

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Naval Command, Control and Ocean Surveillance Center
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OCEAN SURVEILLANCE CENTER
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BACKGROUND

The Toplosky solution (Toplosky and Vogelsong, 1993) for scattering from a rib (or ribs), based on the approach by Stepanishen (1978), is expressed in terms of several response Green's functions. The response Green's functions (or plate cross-admittance functions) are

$$\begin{aligned}\Gamma_0 &\equiv G_F(\kappa_m|\kappa_n) = \frac{k}{2\pi} \int_{-\infty}^{+\infty} Y(\xi) e^{ik\xi(x_m-x_n)} d\xi, \\ \Gamma_1 &\equiv G_M(\kappa_m|\kappa_n) = \frac{k}{2\pi} \int_{-\infty}^{+\infty} ik\xi Y(\xi) e^{ik\xi(x_m-x_n)} d\xi, \\ \Gamma_2 &\equiv G'_M(\kappa_m|\kappa_n) = \frac{k}{2\pi} \int_{-\infty}^{+\infty} (ik\xi)^2 Y(\xi) e^{ik\xi(x_m-x_n)} d\xi,\end{aligned}$$

where k is the wavenumber and $Y(\xi)$ is the admittance of a fluid loaded plate.

$$Y(\xi) = \frac{1}{Z_p(\xi) + Z_a(\xi)},$$

where

$$Z_a(\xi) = \frac{\rho c}{\sqrt{1 - \xi^2}}$$

is the acoustic radiation impedance, with ρ and c the fluid density and sound speed.

$$Z_p(\xi) = i\omega\rho_p h [\Omega^2 \xi^4 - 1]$$

is the impedance of an unstiffened Bernoulli–Euler (thin) plate, where ρ_p and h are the density and thickness of the plate and $\Omega = \frac{\omega}{\omega_c}$. The coincidence frequency

$$\omega_c = \frac{\sqrt{12}c^2}{hc_p},$$

with $c_p = \sqrt{\frac{E}{\rho_p(1 - \nu^2)}}$. E and ν are the Young's modulus and Poisson's ratio of the plate.

Note that the use of a Bernoulli–Euler plate impedance is not justified in the case of a thick plate. When $\Omega > 1$, the results are not valid. Toplosky used it to avoid divergences in his formulation encountered with a Timoshenko–Mindlin plate impedance. The root of the problem was his use of a force dipole line moment, which cannot be supported by a Timoshenko–Mindlin plate (Rumerman, 1979).

INTEGRALS

The Green's functions are evaluated using contour integration. There are branch points at $\xi = \pm 1$ as well as poles in the complex ξ -plane, some on the real axis. The L-shaped Sommerfeld branch cuts are used. The results may be expressed as

$$\Gamma_J = 2\pi i \sum \text{residues}_J -I_{35,J} - I_{26,J},$$

where

$$I_{35,J}(d) = -\frac{k}{\pi} \int_0^1 (ik\xi)^J \frac{\rho c \sqrt{1-\xi^2}}{(\omega \rho_p h)^2 (\Omega^2 \xi^4 - 1)^2 (1-\xi^2) + (\rho c)^2} e^{ikd\xi} d\xi,$$

$$I_{26,J}(d) = i\frac{k}{\pi} \int_0^\infty (-k\xi)^J \frac{\rho c \sqrt{1+\xi^2}}{(\omega \rho_p h)^2 (\Omega^2 \xi^4 - 1)^2 (1+\xi^2) + (\rho c)^2} e^{-kd\xi} d\xi.$$

The advantages of these expressions for numerical integration may be seen. In the case of an oscillatory integrand, the interval is simply $[0,1]$. In the case of an infinite interval, the integrand has exponential damping. The upper limit may therefore be set to a reasonable value (Toplosky uses 8).

NUMERICAL CONSIDERATIONS

Toplosky calculates the Green's functions using simple Romberg integration. This is sufficient at low frequencies. However, at high frequencies, the rational factor has a sharp peak near $\xi = \Omega^{-1/2}$ (width $\Delta\xi \propto \Omega^{-3/2}$), which may cause convergence problems, especially in combination with a rapidly oscillating exponential factor. Variable step size integration, such as in ordinary differential equation solvers, will provide superior performance. For $I_{26,J}$, when $d \neq 0$, the decaying exponential factor negates the significance of the peak.

HIGH-FREQUENCY APPROXIMATIONS

Based on the previous considerations, the integrals may be approximated when $\Omega \gg 1$. Again, note that the Toplosky formulation is not valid when $\Omega > 1$. However, despite this, we will find some useful results, as will be seen later.

We will begin with $I_{35,J}$ first. Change the integration variable to $z = \sqrt{\Omega\xi}$, so

$$I_{35,J}(d) = -\frac{k}{\pi} \frac{\rho c}{\sqrt{\Omega}} \left(\frac{ik}{\sqrt{\Omega}} \right)^J \int_0^{\sqrt{\Omega}} z^J \frac{\sqrt{1 - \frac{z^2}{\Omega}}}{(\omega \rho_p h)^2 (z^4 - 1)^2 \left(1 - \frac{z^2}{\Omega}\right) + (\rho c)^2} \exp\left(i \frac{kd}{\sqrt{\Omega}} z\right) dz,$$

and the peak is now at $z = 1$. Note that $z^4 - 1 = (z^2 + 1)(z + 1)(z - 1) \approx 4(z - 1)$ in the vicinity of the peak, and that z^2/Ω is negligible there as well. Therefore,

$$\begin{aligned} I_{35,J}(d) &\approx -\frac{\rho c k \sqrt{\Omega - 1}}{\pi \Omega} \left(\frac{ik}{\sqrt{\Omega}} \right)^J \int_0^{\sqrt{\Omega}} \frac{\exp\left(i \frac{kd}{\sqrt{\Omega}} z\right) dz}{\left[4\omega \rho_p h \sqrt{1 - \frac{1}{\Omega}}\right]^2 (z - 1)^2 + (\rho c)^2}, \\ &\approx -\frac{k}{\pi} \frac{\exp\left(i \frac{kd}{\sqrt{\Omega}}\right)}{4\omega \rho_p h \sqrt{\Omega}} \left(\frac{ik}{\sqrt{\Omega}} \right)^J \int_{-y_0}^{y_0 \sqrt{\Omega}} \frac{\exp\left(i \frac{kd}{\sqrt{\Omega}} \frac{y}{y_0}\right) dy}{y^2 + 1}, \end{aligned}$$

where $y_0 = 4kh \frac{\rho_p}{\rho} \sqrt{1 - 1/\Omega} = 8\sqrt{3} \frac{\rho_p c}{\rho c_p} \Omega \sqrt{1 - 1/\Omega} \gg 1$ for all practical problems. Thus, taking the integration limits as $\pm\infty$, we finally find

$$I_{35,J}(d) \approx -\frac{\exp\left(-\frac{\rho d}{4\rho_p h \sqrt{\Omega}}\right)}{4\rho_p c h \sqrt{\Omega}} \exp\left(i \frac{\omega_c d \sqrt{\Omega}}{c}\right) \left(i \frac{\omega_c \sqrt{\Omega}}{c}\right)^J.$$

In considering approximations to $I_{26,J}$, we must examine the case $d = 0$ separately.

$$I_{26,J}(0) = i \frac{k}{\pi} \frac{\rho c}{\sqrt{\Omega}} \left(-\frac{k}{\sqrt{\Omega}} \right)^J \int_0^{\infty} z^J \frac{\sqrt{1 + \frac{z^2}{\Omega}}}{(\omega \rho_p h)^2 (z^4 - 1)^2 \left(1 + \frac{z^2}{\Omega}\right) + (\rho c)^2} dz.$$

The development follows that for $I_{35,J}$,

$$I_{26,J}(0) \approx i \frac{k}{\pi} \frac{1}{4\omega \rho_p h \sqrt{\Omega}} \left(-\frac{k}{\sqrt{\Omega}} \right)^J \int_{-y_0}^{\infty} \frac{dy}{y^2 + 1},$$

where $y_0 = 4kh \frac{\rho_p}{\rho} \sqrt{1 + 1/\Omega} = 8\sqrt{3} \frac{\rho_p c}{\rho c_p} \Omega \sqrt{1 + 1/\Omega} \gg 1$. Thus,

$$I_{26,J}(0) \approx i \frac{1}{4\rho_p c h \sqrt{\Omega}} \left(-\frac{\omega_c \sqrt{\Omega}}{c} \right)^J.$$

When $d \neq 0$, the main contribution to $I_{26,J}$ comes from a region where $\xi \ll 1$. Therefore,

$$I_{26,J}(d) \approx i \frac{k}{\pi} \frac{\rho c}{(\omega \rho_p h)^2 + (\rho c)^2} (-k)^J \int_0^\infty \xi^J e^{-kd\xi} d\xi,$$

$$\approx i \frac{1}{\pi d} \frac{\rho c}{(\omega_c \rho_p h \Omega)^2 + (\rho c)^2} \left(-\frac{1}{d}\right)^J (J!).$$

EXAMPLE

Let us consider a 5-cm-thick steel plate (coincidence frequency of 4.7 kHz). The “exact” and approximate results are compared in figures 1 through 9. We find that the approximation of $I_{35,J}$ is accurate (within 10%) for $\Omega > 2$. The approximation of $I_{26,J}$ is accurate for $\Omega > 0.2$. It is this result that makes the analysis worthwhile. Similar results are found for a 2.5-cm-thick steel plate (coincidence frequency of 9.4 kHz).

SUMMARY

Approximations to integrals in the Toplosky rib formulation have been found. Though derived for high frequencies, many are useful over a much broader range. These approximations save a great deal of computation time.

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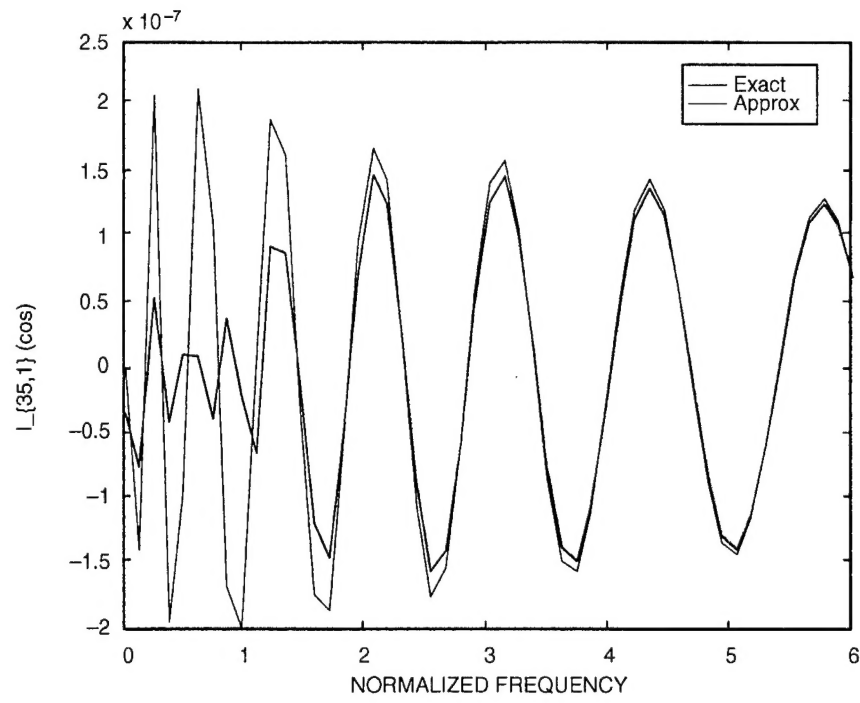


Figure 1. Comparison for $I_{35,1}$ (cosine part).

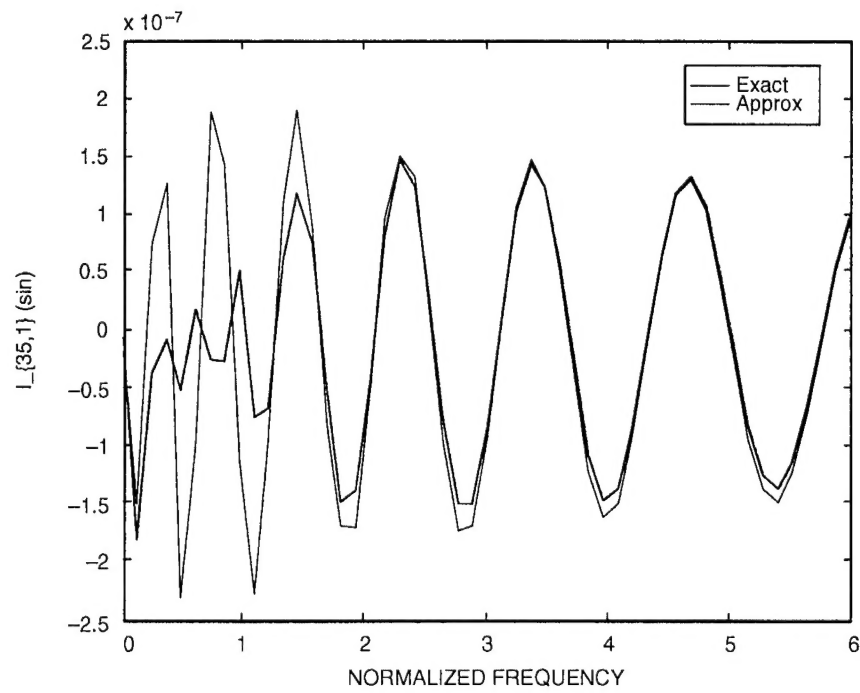


Figure 2. Comparison for $I_{35,1}$ (sine part).

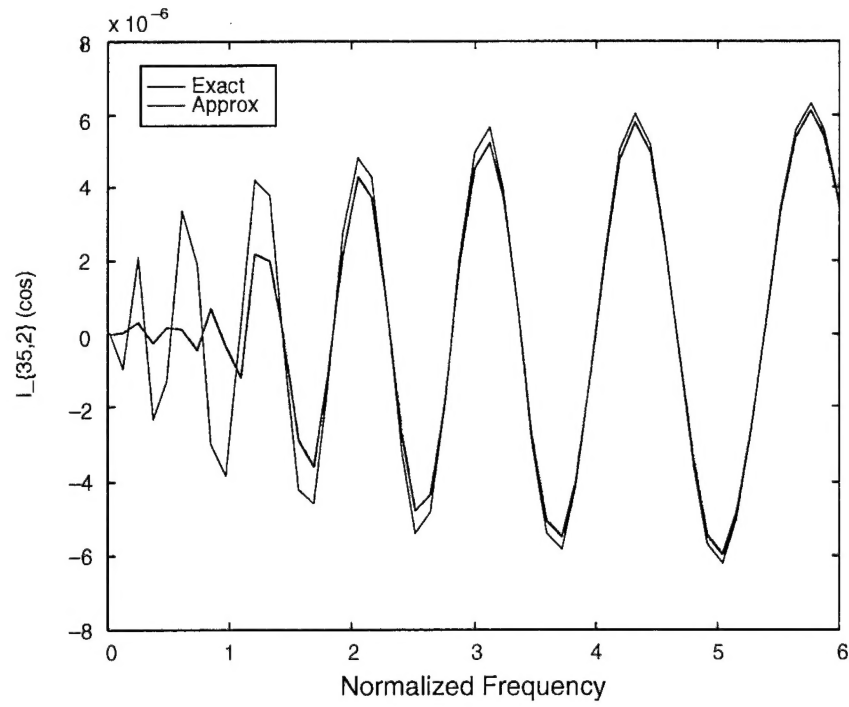


Figure 3. Comparison for $I_{35,2}$ (cosine part).

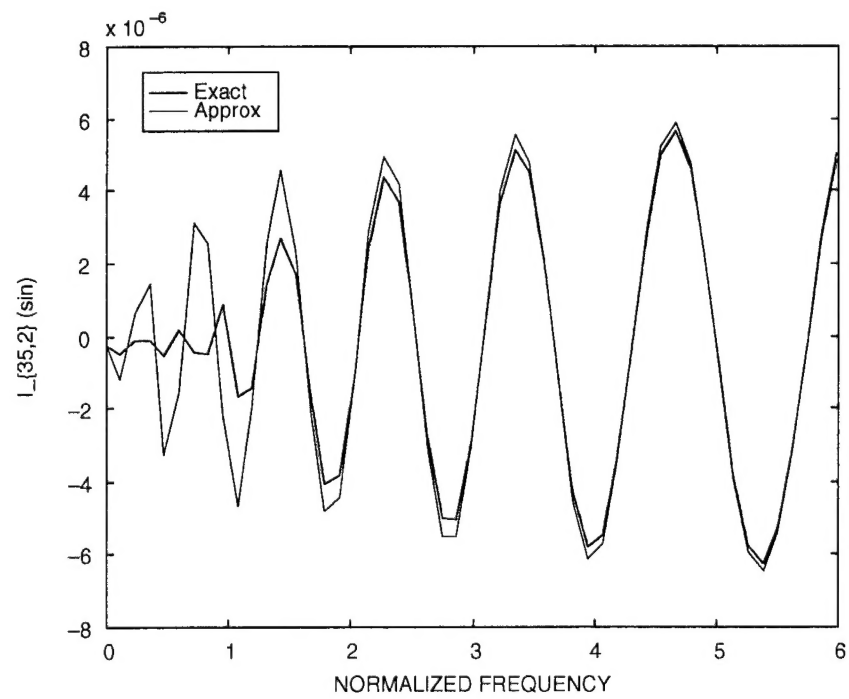


Figure 4. Comparison for $I_{35,2}$ (sine part).

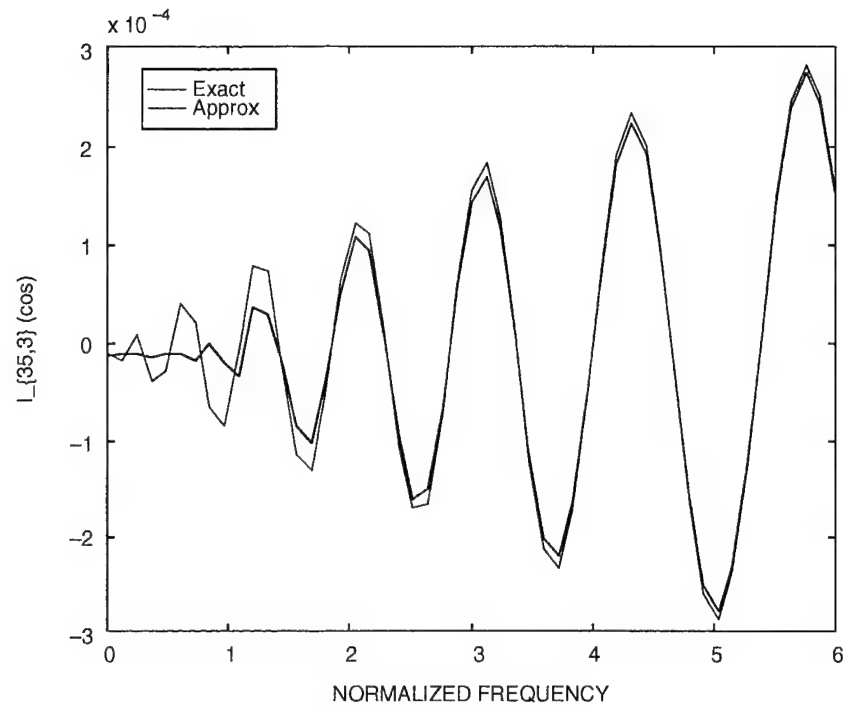


Figure 5. Comparison for $I_{35,3}$ (cosine part).

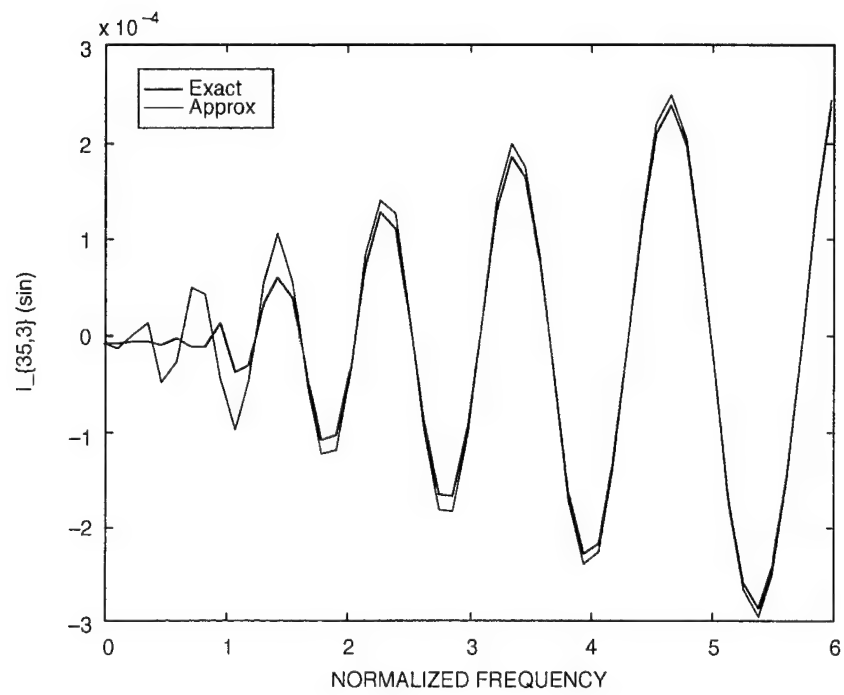


Figure 6. Comparison for $I_{35,3}$ (sine part).

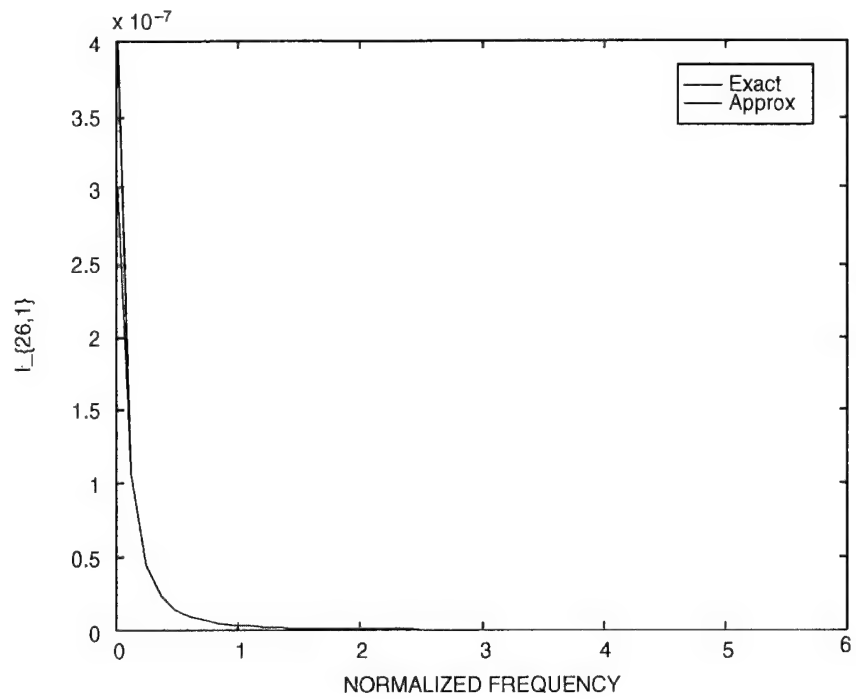


Figure 7. Comparison for $I_{26,1}$.

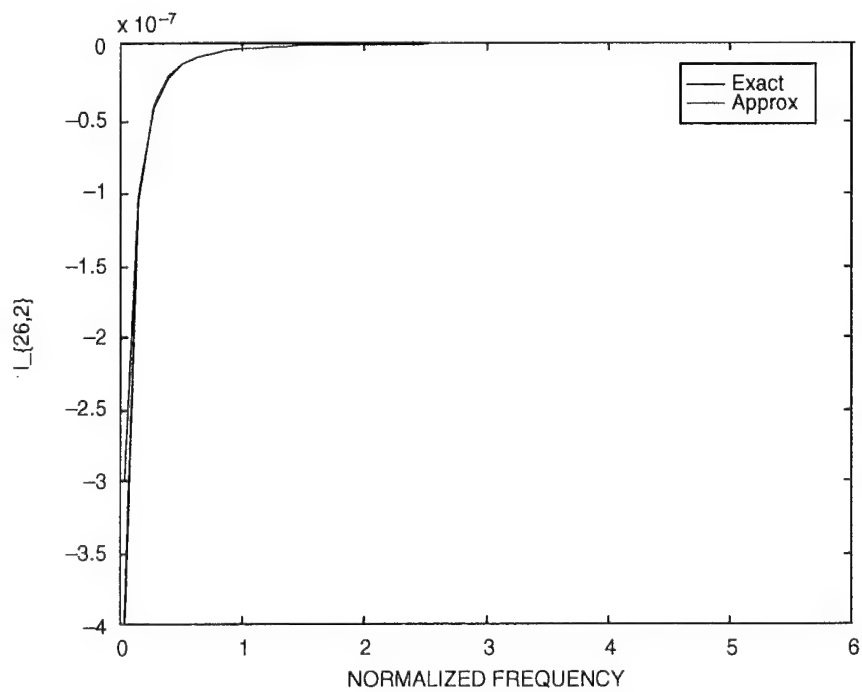


Figure 8. Comparison for $I_{26,2}$.

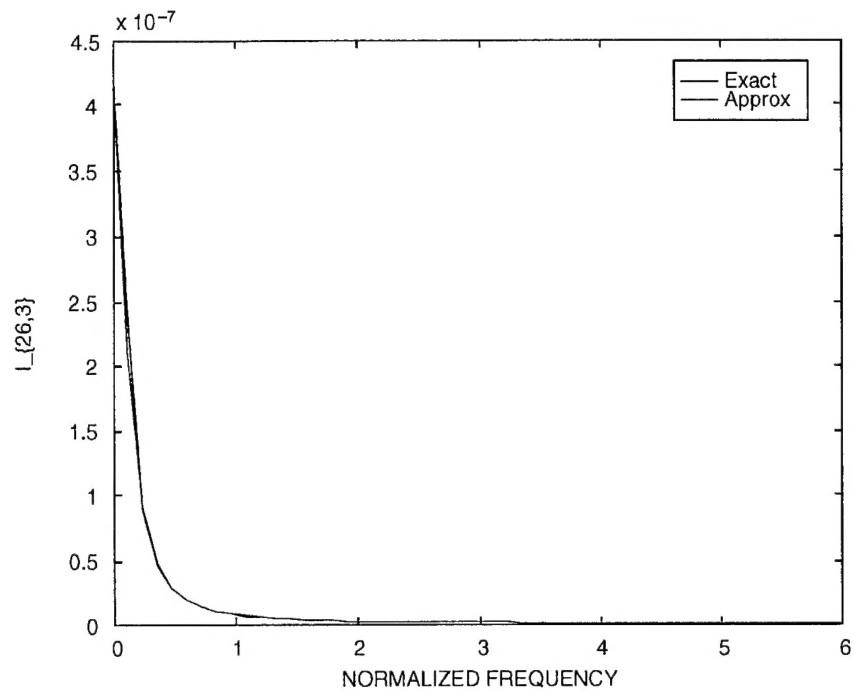


Figure 9. Comparison for $I_{26,3}$.

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